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Spectral Element-Fourier Method for Unsteady Conjugate Heat Transfer in Complex Geometry Flows

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A spectral-element Fourier method (SEFM) is presented for the direct numerical simulation of forced convective heat transfer and conjugate conduction/convection in transitional internal flows in complex geometries. The SEFM is employed for the spatial discretization of the unsteady, incompressible, three-dimensional Navier–Stokes and energy equations. The resulting discrete equations are solved by a semi-implicit method in time treating explicitly the convection operator and implicitly the remaining pressure and viscous contributions. This methodology is illustrated by performing direct numerical simulations to investigate forced convective heat transfer in supercritical self-sustained oscillatory flows and conjugate effects in multimaterial domains. Highly unsteady flows in complex geometries are considered, including modified channels with periodic inhomogeneities such as spanwise rectangular and triangular grooves encountered in electronic equipment and compact heat exchangers.

Introduction

OVER the last decade computational heat transfer (CHT) has experienced exceptional advances due to the improved computer hardware combined with the development of advanced numerical techniques and algorithms. Numerical simulation has emerged as an alternative and, sometimes, as the only approach to analyze in detail complex thermal-fluid phenomena. However, CHT is still at the stage of intensive development, particularly in engineering applications, where most of the problems considered in the past involve significant simplifications regarding geometry, physics and parameter range. Many computational techniques for thermal-fluid problems have been proposed, tested, and refined, mainly for steady flow and time-averaged conservation laws, the latter for modeling transport phenomenon in turbulent flows. Recently there has been an increasing trend toward simulation of more complex thermal-fluid phenomena with a level of complexity that is close to industrial applications. CHT is therefore becoming an emerging field, not only in fundamental research, but also as a design and analysis tool in engineering practice.[1,2]

The ability to simulate complex-geometry and complex-physics flows has grown rapidly in the last two decades because of the effort devoted to computational fluid dynamics (CFD). However, the ability to predict convective heat transfer is still lagging. This is to be expected even in the zeroth-order approach to convective heat transfer where the temperature field is solved in the presence of a known velocity field, i.e., uncoupled momentum and energy equations. Since the velocity field is seldom known, the flowfield needs to be obtained first, making CFD an essential part of CHT. The numerical prediction of convective heat transfer usually requires the combined solution of the velocity and temperature fields that are governed by the equations of conservation of momentum, mass, and energy. Often, simplifying assumptions or models are necessary to make complicated problems tractable. However, if a fairly complete and accurate mathematical description of the main factors affecting the heat transfer phenomena are retained, and the numerical algorithms are suitable to solve the mathematical equation, then the results can be considered an accurate computer simulation of the physical process. Therefore, there are two different issues to consider regarding the analysis of numerical uncertainties: first, the quantifications of the mathematical modeling errors, and, second, the identification and estimation of the numerical errors of the computational scheme used to solve the governing equation modeling the physical phenomenon.

The numerical schemes currently used for the simulation of thermal-fluid phenomena can be broadly classified into Eulerian and Lagrangian schemes with regards to formulation and into spectral, finite difference and finite element techniques with regards to discretization algorithms. These schemes vary in complexity, computing efficiency, numerical accuracy, and flexibility. Each numerical method offers different advantages and limitations for simulating a certain class of transport phenomena. However, no one technique appears to be superior in solving a broad range of problems. Although most of these widely used numerical schemes fall within one of these categories and their differences might have been initially quite clear, recent approaches have combined these traditional schemes, leading to hybrid algorithms with encouraging results for solving complex thermal-fluid phenomena as well as for effective utilization of modern computer architecture.

In the finite element discretization, increased accuracy can be obtained, sometimes at a large computational cost, by employing higher-order interpolating functions while maintaining geometric flexibility. Several hybrid methods have been proposed using high-order polynomial expansions local to finite element, called p or combined h–p finite elements,[3] global elements,[4] and spectral elements.[5,6] The latter is a particularly attractive approach that combines finite elements and spectral schemes. Even though both methods are based on the principle of weighted residuals, the spectral element method utilizes variational projection operators in conjunction with local Chebyshev or Legendre polynomial expansions that exhibit exponential convergence to smooth solutions. Therefore, the spectral element method is a domain decomposition technique that combines the advantages of both globally unstructured and locally structured spatial discretizations. The global decomposition in macro elements provides geometric flexibility, and the local structure permits an efficient high-order approximation by spectral expansions through Chebyshev collocation points.

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For three-dimensional problems, a hybrid spectral-element Fourier method (SEFM) has been proposed to perform direct numerical simulations of the time-dependent, incompressible Navier–Stokes equations in complex geometries with only one homogeneous flow direction.[7] The semidiscrete temporal discretization of the Navier–Stokes equations is achieved through a three-step, time-splitting scheme.[6,7] The first step explicitly accounts for the nonlinear convective contributions and imposes the stability requirement of the scheme. After this step, the problem is reduced to a Stokes formulation that is decomposed into Poisson and Helmholtz elliptic equations for the pressure and viscous terms, respectively. The pressure and velocity are represented by interpolants of the same order and solved in the same mesh. However, the checkrboard pressure field, induced by spurious modes in some finite element techniques, is avoided by a weak imposition of both the incompressibility and boundary conditions on the velocity.[6] Numerical efficiency is achieved through the use of collocation for the nonlinear convective step and variational formulation for the remaining Stokes problem, where the interfacial continuity constraints are imposed naturally via variational statements. This approach results in a weak coupling between the dependent variables in adjacent elements and relative sparse matrices that, implemented together with static condensation, makes this procedure naturally suited for parallel computer implementation. Spectral element methods have also been utilized to simulate unsteady, two-dimensional flows with heat transfer in the presence of fine spatial scales and complex structures,[8] and very good agreement has been obtained between experimental and numerical results of high-frequency, thermofluid phenomena in self-sustained oscillatory flows.[9] The SEFM has been implemented to investigate the structure and evolution of three-dimensional transitional flows in grooved channels,[10] and to simulate laminar and turbulent flows over riblet-mounted surfaces.[11]

In this article, we extend the SEFM to forced convective, time-dependent problems including conjugate conduction/convection heat transfer in multimaterial solid domains. Next, we present the mathematical formulation and discuss the implementation of the SEFM for the solution of the three-dimensional, time-dependent energy equation in complex geometries. Then, we present the time-stepping scheme used for the time discretization and review the fundamental ideas of SEFM for the spatial discretization. Lastly, direct numerical simulations of unsteady conjugate and forced convective transitional flows in complex geometries are presented.

Mathematical Formulation

We consider unsteady, incompressible flows and forced convection in three-dimensional domains with homogeneous spanwise z direction $D = D_f U D_s$. The flow boundaries are composed of solid heat conductive walls, $\partial D_{s-f} = \partial D_f U 2 D_s$, and periodic surfaces, ∂D_p. The mathematical model for a conjugate conduction/convection heat transfer problem with a volumetric heat source is given by a system of partial differential equations for the solid and fluid domains [Eqs. (1–3)]

$$\frac{\partial \boldsymbol{v}}{\partial t} = \boldsymbol{v} \times \boldsymbol{\omega} - \frac{1}{\rho} \nabla \left(P + \frac{\rho v^2}{2} \right) + Re^{-1} \nabla^2 \boldsymbol{v} \text{ in } D_f \quad (1)$$

$$\nabla \cdot \boldsymbol{v} = 0 \text{ in } D_f \quad (2)$$

$$\frac{\partial T}{\partial t} = (Re \cdot Pr)^{-1} \nabla^2 T - \nabla \cdot (\boldsymbol{v} T) + S \text{ in } D \quad (3)$$

where $\boldsymbol{v}(\boldsymbol{x}, t)$ is the velocity field, P is the static pressure, ρ is the density, $\boldsymbol{\omega} = \nabla \times \boldsymbol{v}$ is the vorticity, Re is the Reynolds number, $T(\boldsymbol{x}, t)$ is the temperature field, Pr is the Prandtl number, and S corresponds to the nondimensional volumetric heat source. Buoyancy forces are considered negligible for the range of Reynolds numbers investigated, and viscous dissipation and radiation are likewise neglected in the energy equation.

The boundary conditions for velocity in the Navier–Stokes equation are no-slip along the solid-fluid interfaces, ∂D_{s-f}, and inflow/outflow or periodicity in the streamwise x direction and spanwise z direction (Fig. 1). In the solid region D_s the velocities are zero in the energy equation and no-slip boundary conditions are satisfied. For the forced convective heat transfer equation in the fluid domain D_f, the boundary conditions on the temperature can be either Dirichlet (temperature), Neumann (flux), mixed Robin (heat transfer coefficient), periodicity or continuity of temperature and heat flux for conjugate problems. The appropriate thermal boundary conditions will be discussed in the context of the individual applications presented in the later sections.

Simplified thermal modeling procedures often reduce a forced convective problem to solving the heat equation within the solid domain D_s and imposing the convective effects of the fluid through a heat transfer boundary condition at the solid–fluid interface s–f [Eq. (4)]

$$h(T_{s-f} - T_{\text{ref}}) = -k_s \frac{\partial T_s}{\partial n} \quad (4)$$

where h is the convective heat transfer coefficient is a proportionality constant that models the ability of the fluid to remove heat at the solid-fluid interface. The proper reference temperature T_{ref} is problem-dependent and can be chosen as the ambient, bulk, adiabatic, or inlet temperatures. Reformulation of the problem necessitates that the heat transfer coefficient be obtained either empirically or numerically by solving the coupled system of equations (1–3) only in the fluid domain D_f and imposing thermal boundary conditions along the solid–fluid boundary. This implies that the effect of heat conduction within the solid is replaced by idealized boundary conditions that assume the heat flow paths in the solid and decouple the resistance associated with conduction within the solid and convection within the fluid. However, in conjugate problems, neither the temperature nor the heat flux at the solid–fluid interface can be prescribed accurately a priori, especially in systems that involve intense heat transfer, multimaterial solid domains, and localized heat generation.[12] Therefore, the convective boundary condition, as previously described, may not provide accurate predictions. The appropriate thermal boundary conditions for conjugate conduction/convection are continuity of heat flux and temperature at the solid–fluid interface and are termed boundary conditions of the fourth kind[13] [Eqs. (5a) and (5b)]

$$k_s \frac{\partial T_s}{\partial n} = k_f \frac{\partial T_f}{\partial n} \quad (5a)$$

$$T_{s_{s-f}} = T_{f_{s-f}} \quad (5b)$$

Different numerical techniques have been implemented to investigate conjugate conduction/convection and to demonstrate the effect of conjugation on thermal performance characteristics. However, most of the situations analyzed have

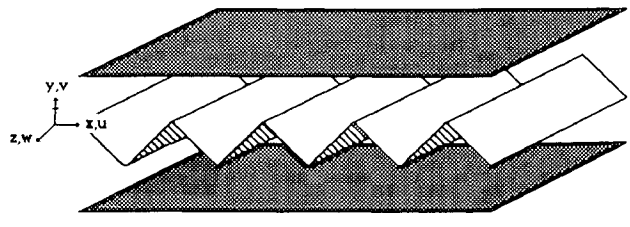


Fig. 1 Schematic of a compact heat exchanger channel geometry with V-grooved riblets.

been steady state with homogeneous solid domains and simple geometries. A two-dimensional steady conjugate study in a laminar boundary layer with a heat source at the solid–fluid interface was conducted using finite differences.[14] The time-dependent conjugate behavior of a semi-infinite slab exposed to uniform surface heating was studied using the unsteady surface element method.[15] The two-dimensional conjugate behavior of hydrodynamically, fully developed, laminar flow through a circular tube with thick walls and a finite heated length was investigated using a finite volume approach.[16] A semi-analytical approach that utilizes an integral formulation for the fluid domain and a finite volume formulation for the solid domain was successfully developed to study plates with discrete heat sources, which model surface-mounted electronic packages.[17] Mixed laminar convection from local heat-generating components was also studied using a simpler-based approach.[18] Time-dependent studies of multimaterial, local heat generating configurations using the spectral element method were conducted by Nigen and Amon[19,20] for both laminar and transitional Reynolds numbers. This investigation contrasted thermal behavior characteristics for conjugate and convection-only representations of a simulated electronic package and demonstrated the significance of including time-dependency and conjugation.

Temporal Discretization

To discretize in time the Navier–Stokes equation [Eq. (1)], we use a semi-implicit fractional step method,[5–7] with explicit treatment of the nonlinear convective terms, and implicit methods for the pressure and viscous contributions. To sustain the flow, this equation of conservation of momentum should include a nonzero pressure gradient in the direction of motion.

For the energy equation [Eq. (3)] we use a similar semi-implicit time-stepping scheme with two steps. The first step is an explicit third-order Adams–Bashforth step for the convective and source volumetric heat generation terms, and the second step is an implicit Crank Nicolson step for the conductive terms. The semi-discrete equations for $T^n(\boldsymbol{x}) = T(\boldsymbol{x}, n\Delta t)$ are then

$$\hat{T}^{n+1} - T^n = -\Delta t \sum_{q=0}^{2} \beta_q \nabla\cdot(\boldsymbol{v}T)^{n-1} + S \qquad (6)$$

$$T^{n+1} - \hat{T}^{n+1} = -\frac{\Delta t}{2Re\cdot Pr}[\nabla^2(T^{n+1} + T^n)] \qquad (7)$$

where $\beta_0 = 23/12$, $\beta_1 = -16/12$, and $\beta_2 = 5/12$ are the Adams–Bashforth coefficients. All boundary conditions are imposed on the conduction step [Eq. (7)].

The numerical approach consists of integrating the continuity and Navier–Stokes equations for the fluid portion of the domain and then integrating the energy equation for both the solid and fluid domains. This procedure is iterated in time until either an asymptotically-steady or time-periodic state is reached.

Spatial Discretization

We consider flows for which the time-averaged velocities and temperatures vary only in the (x, y) plane and are constant along the spanwise z direction. Although the geometry remains effectively two dimensional (Fig. 1), the instantaneous changes of the unknown velocity, pressure, and temperature in the spanwise direction are resolved. This assumption of homogeneous geometry in the spanwise z direction enables discretization of the computational domain with spectral elements in the (x, y) plane and spectral Fourier expansions along the z direction.[7] After expanding the dependent variables in the z direction by Fourier truncated series, the dependent variables in the (x, y) spatial domain are discretized by isoparametric spectral elements and expanded in tensor product

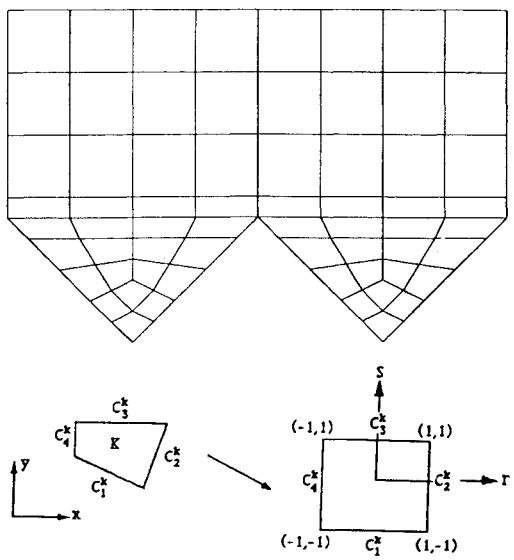


Fig. 2 a) Macro-element subdomain decomposition of the (x, y) plane and b) isoparametric mapping of a typical macro-element.

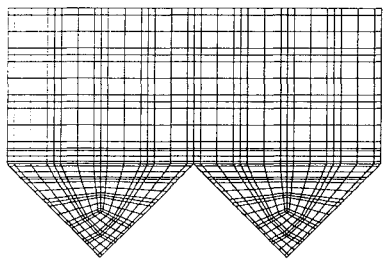
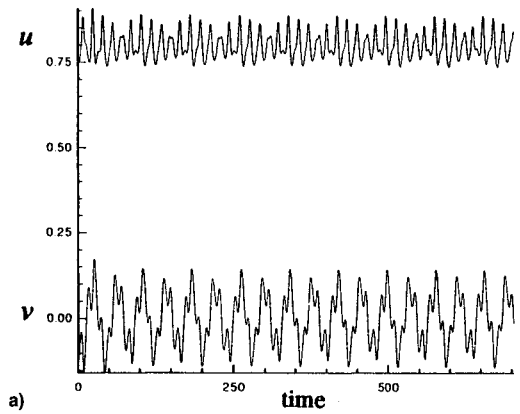


Fig. 3 Computational domain and spectral element mesh.



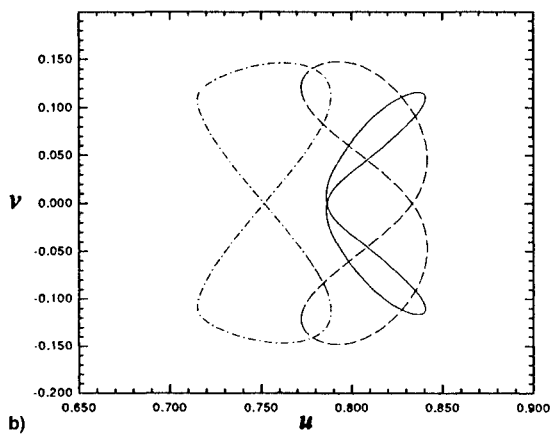


Fig. 4 a) Time history of characteristic velocity components and b) phase planes of v velocity vs u velocity.

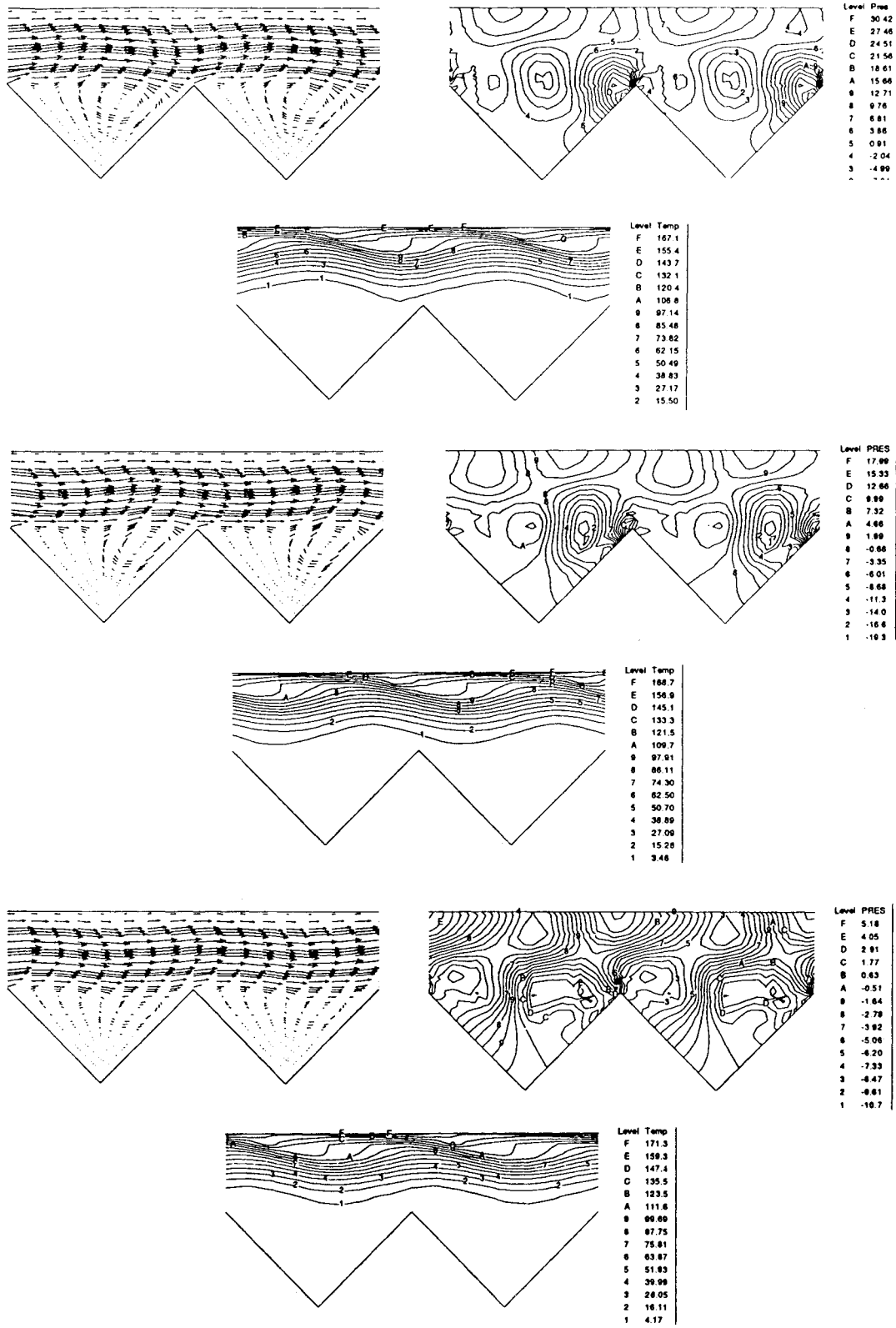


Fig. 5 Instantaneous velocity vectors, pressure contours, and isotherms during one cycle of the self-sustained oscillatory flow.

bases in terms of local elemental coordinates (r, s) through Gauss–Lobatto Chebyshev collocation points.[6] A typical elemental discretization for the compact heat exchanger geometry with V-grooved riblets (Fig. 1) is shown in Fig. 2, and the distribution of the collocation points is shown in Fig. 3. A variational formulation is used, thereby, automatically satisfying continuity of the dependent variables conditions across elemental boundaries. This approach does not require a patching technique, which is usually needed to obtain interfacial continuity in multidomain spectral formulations.[4]

The numerical accuracy of the spectral element simulations can be improved by increasing skillfully the total number of macro-elements (h refinement) obtaining algebraic convergence, by increasing the order of the local expansions (p refinement) achieving nearly-exponential convergence, or by performing simultaneously h and p mesh refinements (h–p

refinement). Adequate mesh resolution is verified by comparing the thermal and flow characteristics using different order of local expansions and/or macro-elemental discretizations. The control of spatial resolution and the high degree of accuracy associated with this technique make it well-suited for studying conjugate problems with localized heat generation and multimaterial solid domains, especially those with large variation in material properties.

Numerical Examples

In this section we present direct numerical simulations of forced convective heat transfer and conjugate conduction/convection in transitional flows in modified channels with periodic inhomogeneities such as V-grooved riblets and grooved channels. The geometric parameters are selected to excite and sustain the normally damped Tollmien–Schlichting channel modes.[10] As a result, traveling waves are observed at relatively low Reynolds numbers, inducing self-sustained oscillatory flows that significantly enhance mixing. The oscillatory separated flow induces large-scale convective patterns that are responsible for substantial heat transfer enhancement and reduction in the pumping power required to achieve a given heat transfer performance.[8]

Above a critical Reynolds number, these flows bifurcate from a steady to a time-periodic, self-sustained oscillatory flow. The nature of these oscillations is indicated in Fig. 4a by a time-history plot of the velocity components at a characteristic point of the domain, and in Fig. 4b by the corresponding phase space planes of the crosswise v velocity vs the streamwise u velocity at three different points of the computational domain. The power spectrum of the velocity reveals the existence of two fundamental frequencies, one of which is very close to the least-stable Tollmien–Schlichting frequency. The transition process from a laminar to a chaotic state in converging-diverging channel flows occurs through series of successive supercritical Hopf bifurcations,[21] and a scenario similar to the Ruelle–Takens–Newhouse scenario at the onset of chaos is verified.[22]

Figure 5 shows the temporal evolution of the thermal-flow structure in a sequence of three time frames during one period of the self-sustained oscillation in terms of velocity vectors, pressures, and temperature fields. Free shear layers span the riblet openings and separate the mainstream from the recirculating flow region in the riblet cavities. Periodic interruptions of the shear layers cause their instabilities to project energy onto the normally damped Tollmien–Schlichting modes in order to sustain them. The waviness of the channel flow induces the ejection of the vortices from the riblet cavities when the low-pressure region associated with the crest of the wave passes over it. Subsequently, mainstream fluid fills the riblet cavity and a new vortex grows and protrudes into the mainstream flow, thus enhancing flow mixing.

The instantaneous isotherms in Fig. 5 depict relatively uniform temperatures in the riblet regions. This is indicative of the convective mixing between the riblet and channel flows due to the self-sustained oscillations. Figure 5 also shows the creation of thin thermal boundary layers on the channel top wall induced by the thermal traveling wave. These results suggest that at moderate Reynolds numbers, the riblet surfaces should provide a better performance than flat-walled passages when comparisons of heat transfer as a function of fluid pumping power are performed.[23] The channel with V-grooved riblets offers significantly higher heat transfer for a given pumping power, by a factor of approximately 3.5, compared to a flat channel in the laminar-transitional regime.[24] The passive flow destabilization, induced by the spatially periodic disturbances introduced by the riblets, sustains the oscillatory flow, the Tollmien–Schlichting traveling waves and the vortex ejections that are the flow mechanisms responsible for enhancing the overall mixing and, hence, improving the heat transfer performance.

As our last example, we consider a periodic grooved-channel geometry that is a model frequently used to simulate surface-mounted electronic components on printed circuit boards. Figure 6 shows a schematic of the grooved channel with the configuration of the electronic components, dimensions, and material composition. Numerical simulations of the time-dependent conjugate conduction/convection heat transfer are performed to ascertain the influence that conjugate effects have upon the convective heat transport. The conjugate results are also contrasted with those results obtained from simulating only the fluid domain for the same geometry with uniform heat-flux boundary conditions along the grooved wall. Instantaneous isotherms in the fluid region are shown in Fig. 7, depicting the wavy character of the isotherms in the bypass region that is a consequence of the traveling waves. The complex supercritical flow structure induces a time-repetitive sequence of convective exchange between the groove and the bypass regions. The periodic disruption of the shear layer coupled with the separation flow phenomenon at the downstream groove corner results in projection of the hot fluid from the downstream component face into the bypass flow.

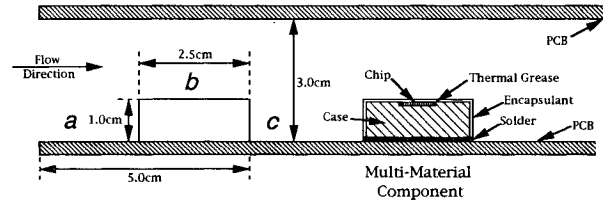


Fig. 6 Schematic of grooved channel and electronic component configuration.

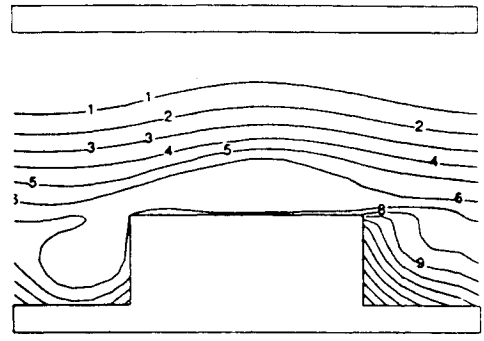


Fig. 7 Instantaneous isotherm plot in the fluid region of the grooved channel.

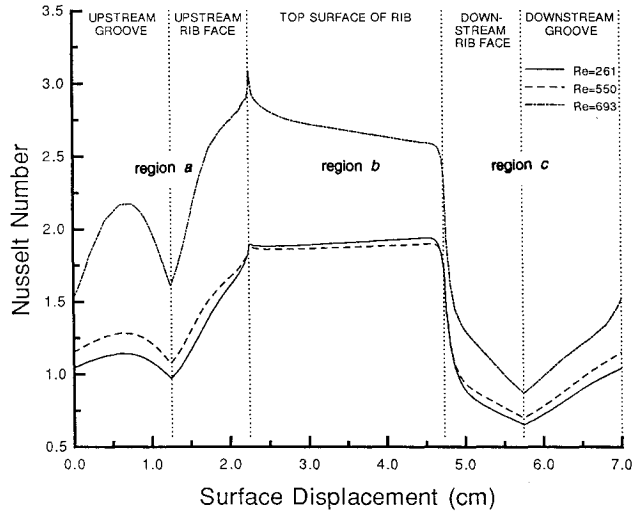


Fig. 8 Time-averaged Nusselt number distribution along the grooved-channel wall, for subcritical (Re = 261, 550) and supercritical (Re = 693) flows for convection-only simulations.

Figure 8 shows the distribution of the time-averaged Nusselt number along the bottom wall of the grooved channel for the fluid-only simulations and three Reynolds numbers *Re* encompassing both the subcritical (*Re* = 261, 550) and the supercritical flow regimes (*Re* = 693). Dividing the grooved surface into three regions (*a*, *b* and *c*, Fig. 6) enables the visualization of the distinct behaviors. The heat transfer coefficient in region *b* remains constant with increasing *Re* in the subcritical regime, which is indicative of fully developed, laminar transport characteristics, and increases dramatically as *Re* is increased beyond the critical value. The heat transport within the groove (regions *a* and *c*) is dictated by the position and rotational strength of the groove vortex, deteriorating in region *c* and improving in region *a* as *Re* is increased in the laminar subcritical flow regime. Once the supercritical regime is reached, the heat transfer coefficient increases significantly along all the grooved walls. This increase is caused by the mixing between the cooling fluid from the bypass region and the groove fluid and by the periodic sequence of vortex growth and ejection from the groove.

In the conjugate conduction/convection simulations, heat is generated within the chip (Fig. 6), conducted through the electronic component and convected into the cooling fluid. The local flow characteristics in the grooved channel are the same as in the convection-only simulations with uniform heat-flux boundary conditions. The time-averaged heat flux distribution for the conjugate simulation is shown in Fig. 9 for subcritical (*Re* = 261) and supercritical (*Re* = 590) flow regimes. In the subcritical regime, the intact shear layer separating the groove and bypass flows precludes convective exchange of fluid. Therefore, the fluid recirculating within the groove, which is heated along the upstream and downstream component faces, becomes hotter than the local surface temperature at the groove bottom. This temperature gradient causes heat to be transferred from the fluid to the groove surface, producing negative heat fluxes in regions *a* and *c* (Fig. 9). The bypass flow convectively cools the fluid within the groove when the shear layer is disrupted and the flow becomes supercritical and oscillatory. The heat flux distribution along region *b* is affected by the growth of the thermal boundary layer and the location of the heat generation, leading to an exponential decline in the downstream direction with a local rise directly over the location of the chip.[20] Therefore, time-dependent forced convective transport phenomena, on complex, multimaterial solid domains with conjugate conduction/convection, exhibit different thermal performance than that obtained by assuming uniform heat-flux boundary conditions. These results illustrate the importance of accounting for conjugate conduction/convection effects in the design and analysis of heat exchanger configurations such as those found in the cooling of electronic components.

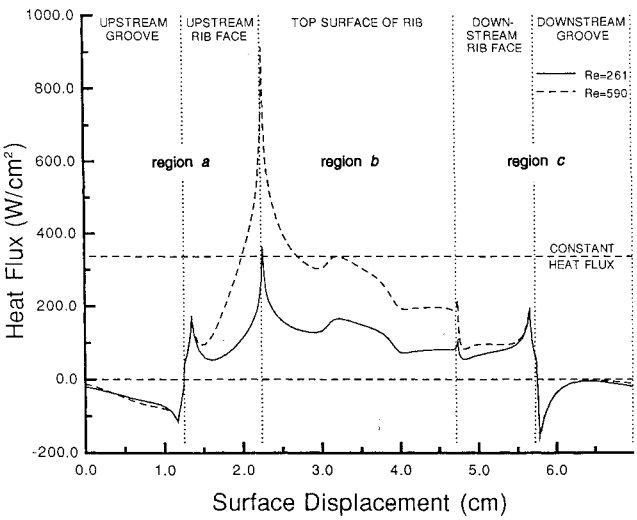


Fig. 9 Time-averaged heat flux distribution along the solid–fluid interface for subcritical (*Re* = 261) and supercritical (*Re* = 590) flows for conjugate conduction/convection simulations, and constant heat flux for the convection-only simulations.

Acknowledgments

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